Aggregation of Multivariate Extremes

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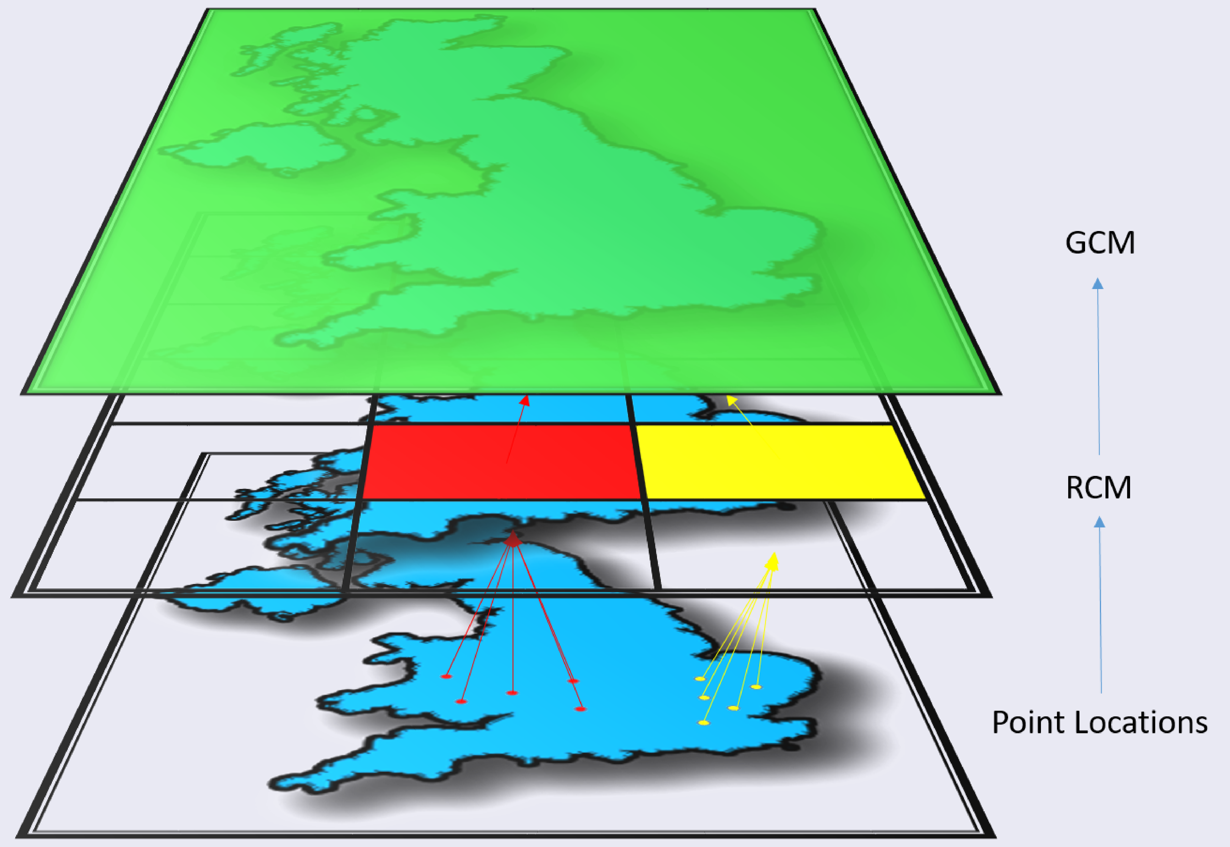
Motivation

- **Fluvial flooding** is caused by sustained excessive rainfall over **extended periods of time** and **spatial catchment areas**.
- The **Met Office** develops **Global Climate Models**, which forecast precipitation. These are run on spatial grid boxes at discrete time steps.
- We are interested in the behaviour of extreme rainfall over different spatial and temporal aggregations, specifically the return levels.



The return-levels for increasing aggregation should **monotonically change** with aggregation level. However, individual estimates do not always follow this pattern. Further analysis is needed to determine the underlying structure.

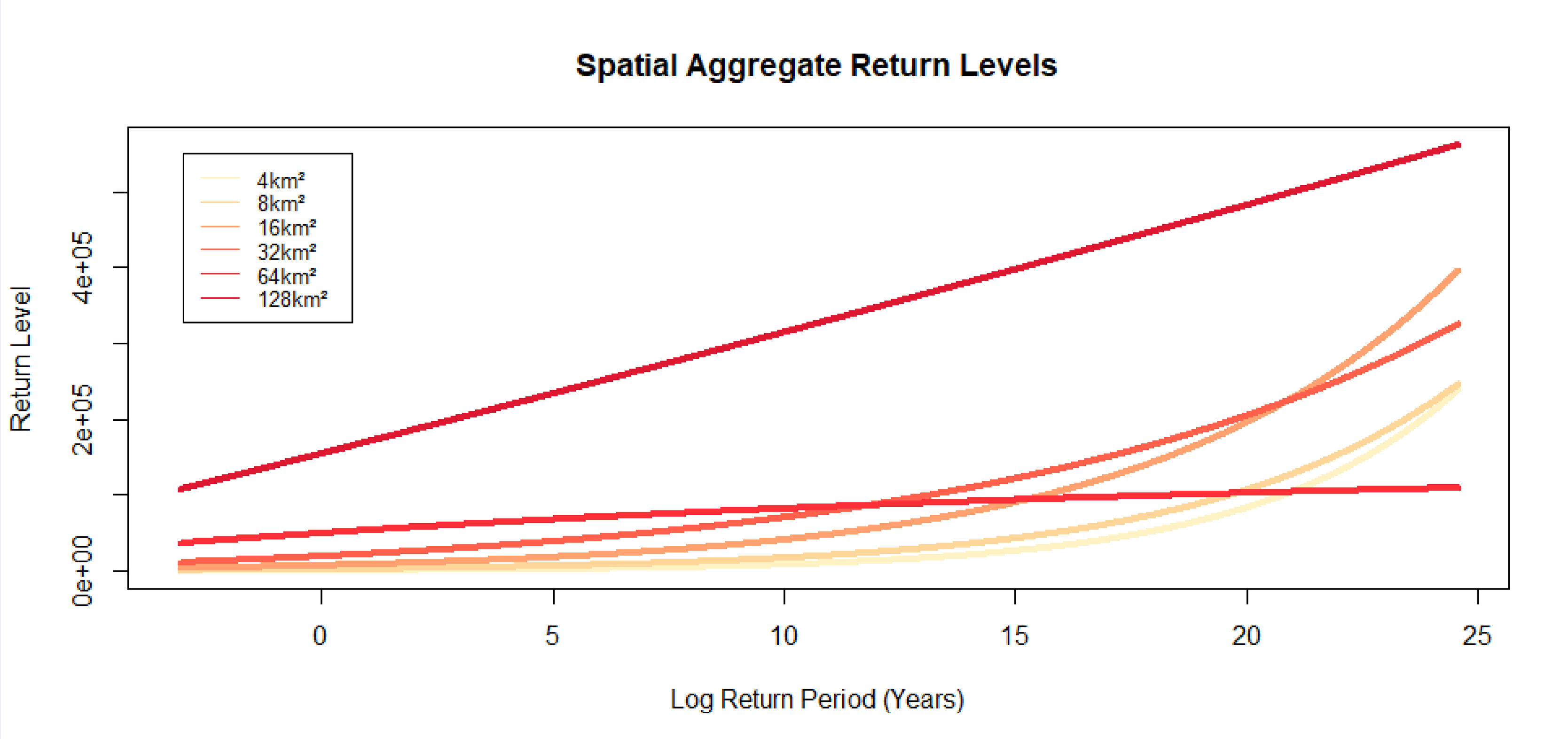


Figure 1: Return level curves for decreasing levels of spatial resolution. Data is gridded precipitation, JFD 1979 - 2015, located in the North-West of England.

Aims/Goals

- Establishing a relationship between tail behaviour of local variables and aggregations.
- Incorporating different **extremal dependence** structures.

Set-up

- Variables, X_i, are assumed to have GPD tails with uncommon scale and shape parameters.
- Dependence between variables is induced using copulas.
- We are interested in the tail of Y, where Y is the average of all X_i.
- Y has GPD tails with scale σ_Y and shape ξ_Y, which are greatly affected by the marginal shape parameters and extremal dependence structure.

Extremal Dependence

- **Extremal dependence** is the tendency for extremes to occur together.
- For **multivariate extremes**, this is often represented by pairwise measures between variables.

Extremal dependence is measured using pairwise summary measures, χ and η, where

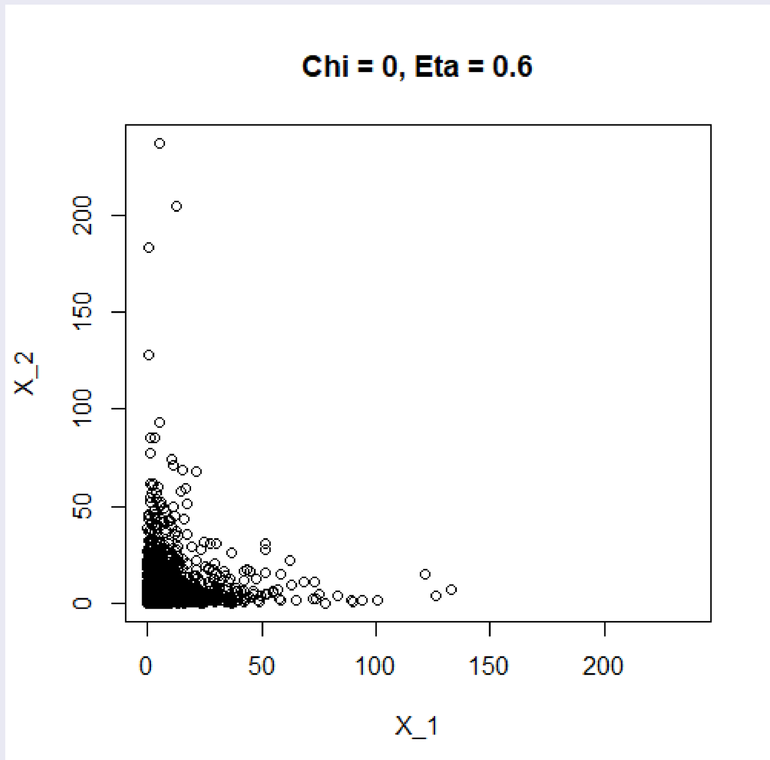
$$\chi := \lim_{u \to 1} \Pr\{F_1(X_1) > u | F_2(X_2) > u\}, \quad X_i \sim F_i$$

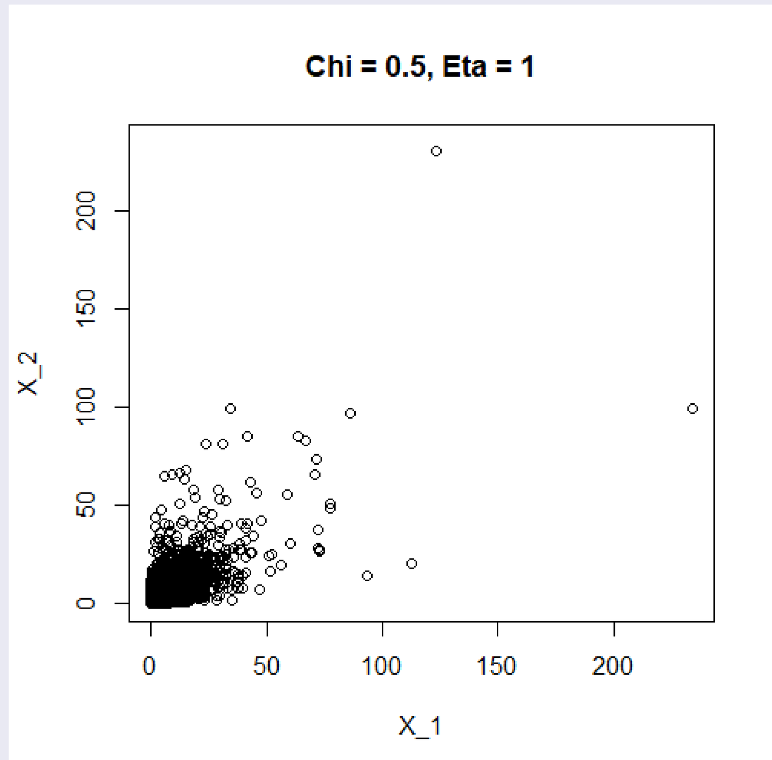
and

$$\Pr\{F_1(X_1) > u, F_2(X_2) > u\} = L\left(\frac{1}{1-u}\right)(1-u)^{1/\eta} \quad \text{as} \quad u \to 1,$$

with $L(x)$ a slowly-varying function, and $\chi \in [0,1]$, $\eta \in [1/2, 1]$. The pair of measures characterises extremal dependence:

- $(\chi > 0, \eta = 1)$ corresponds to an **asymptotically dependent** copula
- $(\chi = 0, \eta < 1)$ corresponds to an **asymptotically independent** copula





We consider different copulas, as these determine the dependence structure:

- **asymptotically dependent (AD)** - extreme value copula, perfect dependence
- **asymptotically independent (AI)** - inverted extreme value copula, multivariate Gaussian ($\rho > 0$), independence

Note that d-dimensional versions of both measures exist. For example, we have $\eta_d \in [1/d, 1]$, where $1/d$ corresponds to independence.

Results - Homogeneous Shape

We have recently derived the distribution of the sums of $d-$dimensional random variables with the 5 different copula types and **marginal tails** with homogeneous shape and scale parameters. The GPD tail of Y has scale and shape parameters (σ_Y, ξ_Y):

- $(\sigma_Y, \xi_Y) = (\eta_d \sigma, \eta_d \xi)$ if $\xi \leq 0$.
- $(\sigma_Y, \xi_Y) = (K\sigma, \xi)$ if $\xi > 0$.

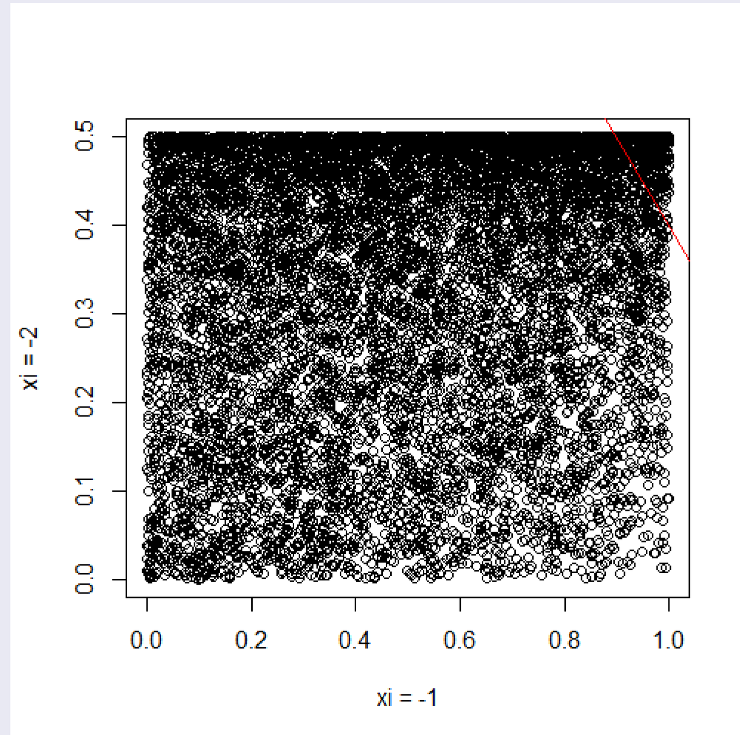
K is not a function of η_d and depends on the margins and the dependence structure. For the limiting cases, we have shown that

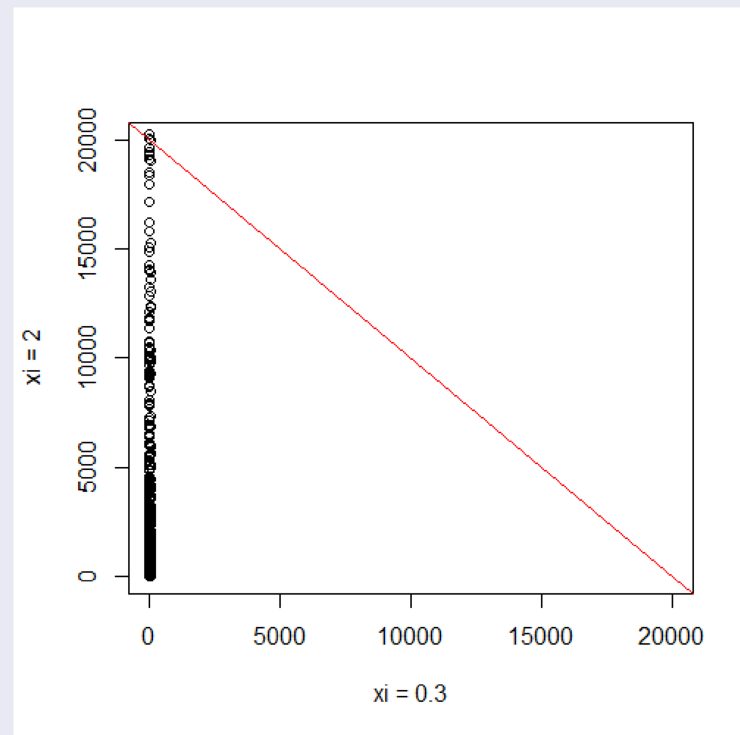
- $K = 1$ for perfect dependence.
- $K = d^{\xi - 1}$ for independence.

Results - Heterogeneous Shape

We have also considered the effect of **combinations** of marginal shapes on the tail of Y. Regardless of the dependence structure,

- $\xi_Y = \max_i\{\xi_i\}$ if the maximum is positive.
- ξ_Y is a weighted combination of all marginal shape parameters, if the maximum is negative.





If the tail of Y is **bounded**, all marginals must contribute to the behaviour. If the tail of Y is **unbounded**, only the heaviest marginal tails contribute.

References and Acknowledgements

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- Ledford, A. W., and Tawn, J. A. (1996). Biometrika. *Statistics for Near Independence in Multivariate Extremes.*

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