Generative AI for Statistics of Joint Metocean Extremes

Jordan Richards
Joint work with Ed Mackay, Callum Murphy-Barltrop, and Phil Jonathan

School of Mathematics and Maxwell Institute for Mathematical Sciences, University of Edinburgh, UK

22/07/2026

Metocean multivariate extremes

- 31-year hindcast of hourly values for a site in Celtic sea ($n \approx 270000$).
- $d = 5$, x- and y-components of wind speed (U) and wave height (H), and (log)-wave period (T_m).
- Ocean storms driven by non-convex combinations of marginal "extremes" with physical limits ($s_{\max}$)

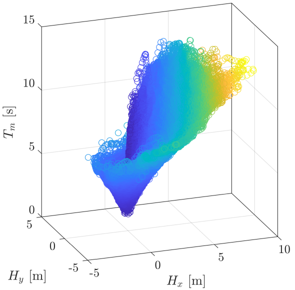
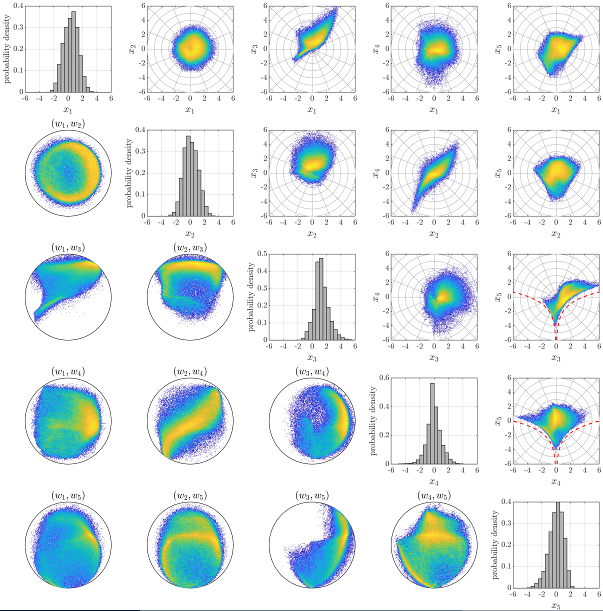


Figure: x-y components of H_s and T_m. Colours indicate the value of $H_s = \sqrt{H_x^2 + H_y^2}$ (low, high), which relates to wave steepness $s = 2\pi H_s/(gT_m^2)$.

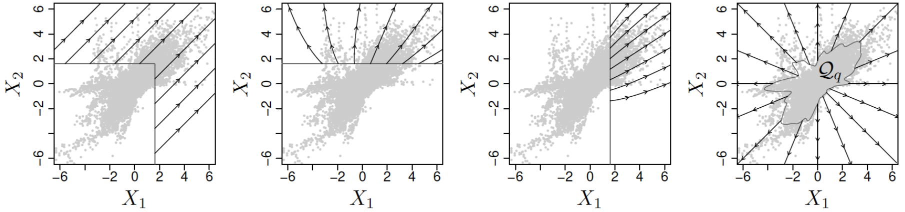
Metocean multivariate extremes



Multivariate extremes

Traditional models capture one type of multivariate extreme or are limited in the **strength/class** of extremal dependence they can capture.

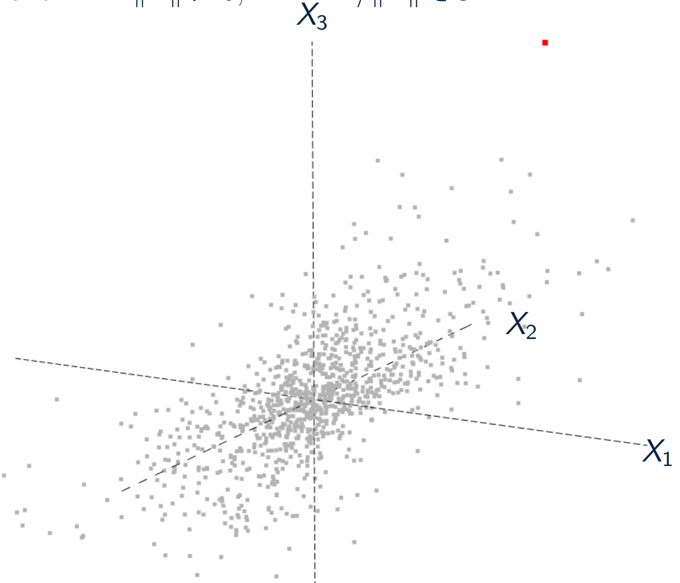
We reproduce all joint extremes, rather than one variable being large, using a **generative model**.



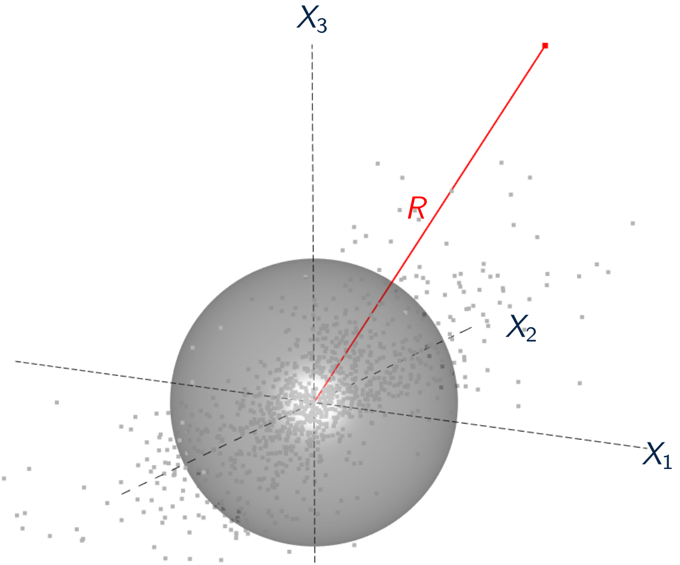
Papastathopoulos, I., De Monte, L., Campbell, R., & Rue, H. (2023). Statistical inference for radially-stable generalized Pareto distributions and return level-sets in geometric extremes. arXiv:2310.06130.

Angular-radial representations

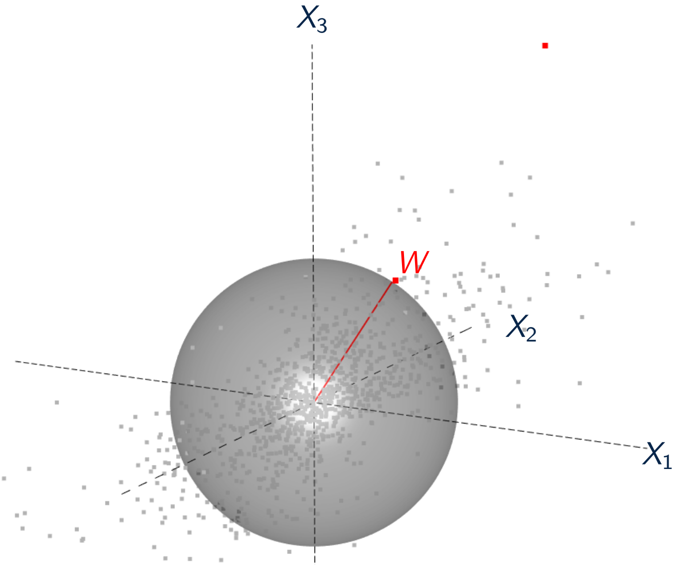
Let $\mathbf{X} \in \mathbb{R}^d$ and $R := \|\mathbf{X}\| > 0$, $\mathbf{W} := \mathbf{X}/\|\mathbf{X}\| \in \mathcal{S}^{d-1}$.



Angular-radial representations



Angular-radial representations



Angular-radial representations

The joint density of $(R, \mathbf{W})$ can be written as

$$f_{\mathbf{R},\mathbf{W}}(r, w) = f_{\mathbf{W}}(\mathbf{w}) f_{R|\mathbf{W}}(r|\mathbf{w}). \tag{1}$$

The Semi-Parametric Angular-Radial model (SPAR; Mackay and Jonathan, 2023) replaces $f_{R|\mathbf{W}}(r|\mathbf{w})$ with a conditional GP model **when r is big**, i.e.,

$$f_{R|\mathbf{W}}(r|\mathbf{w}) \approx \zeta f_{\mathrm{GP}}(r - u(\mathbf{w}); \tau(\mathbf{w}), \xi(\mathbf{w})), \tag{2}$$

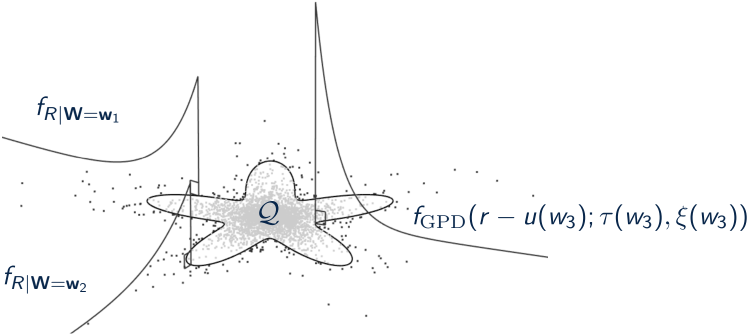
for large $r > u(\mathbf{w})$ and where $\zeta := \Pr(R > u(\mathbf{w}) | \mathbf{W} = \mathbf{w})$ is fixed.

- Mackay et al. (2025) models $(u(\mathbf{w}), \tau(\mathbf{w}), \xi(\mathbf{w}))$ via **deep extremal regression**.

Mackay, Jonathan (2023). Modelling multivariate extremes through angular-radial decomposition of the density function. arXiv.

Mackay, Murphy-Barltrop, Richards, Jonathan (2025). Deep Learning Joint Extremes of Metocean Variables Using the SPAR Model. JOMAE.

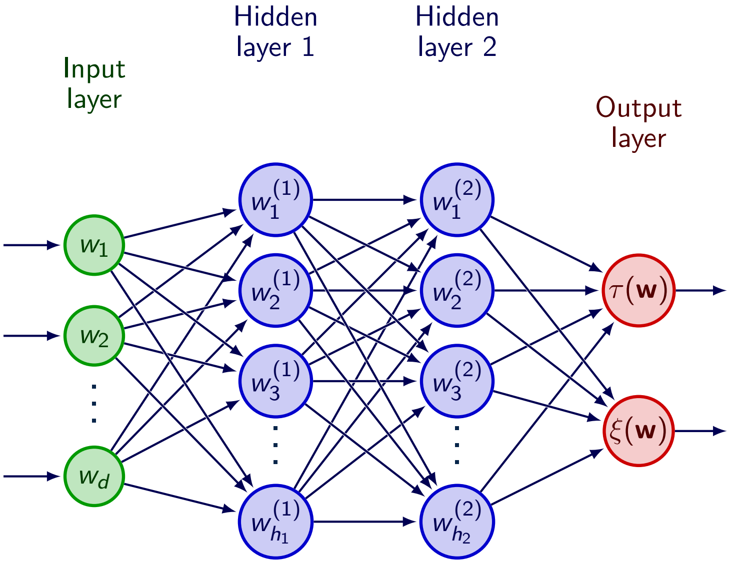
Angular-radial representations



Quantile set $\mathcal{Q} := \{\mathbf{x} : \|\mathbf{x}\| < u(\mathbf{x}/\|\mathbf{x}\|)\}$ with $\Pr\{\mathbf{X} \in \mathcal{Q}\} = 1 - \zeta$.

De Monte, L., Huser, R., Papastathopoulos, I., & Richards, J. (2025). Generative modelling of multivariate geometric extremes using normalising flows. arXiv:2505.02957.

Deep regression for extremes



Angular-radial representations

Recall

$$f_{R,\mathbf{W}}(r,\mathbf{w}) \approx f_{\mathbf{W}}(\mathbf{w})\zeta f_{\mathrm{GP}}(r-u(\mathbf{w});\tau(\mathbf{w}),\xi(\mathbf{w})), \quad r > u(\mathbf{w}). \tag{3}$$

With estimates of $(u(\mathbf{w}),\tau(\mathbf{w}),\xi(\mathbf{w}))$, simulation from $f_{R|\mathbf{W}}$ is simple.

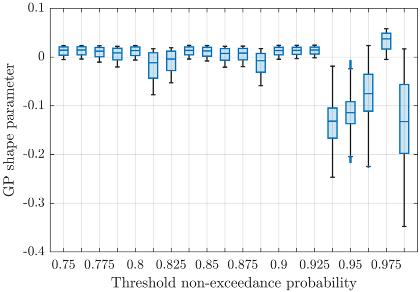
In combination with a model for $f_{\mathbf{W}}$ (see, e.g., Wessel et al., 2025), we can simulate from $(R,\mathbf{W})$ by drawing from (3) with prob. ζ and, otherwise, from the corresponding *non-extreme* empirical distribution.

Inference uses maximum likelihood estimation via *stochastic gradient descent* with **Keras for R**.

Wessel, Murphy-Barltrop, Simpson (2025). Generative machine learning for multivariate angular simulation. Extremes.

Model

- For $(\tau(\mathbf{w}), \xi(\mathbf{w}))$, use MLP with 3 Layers, 16 neurons. Runs on standard laptop.
- $\zeta = 0.1$ via threshold stability plots.
- KDE for angles.



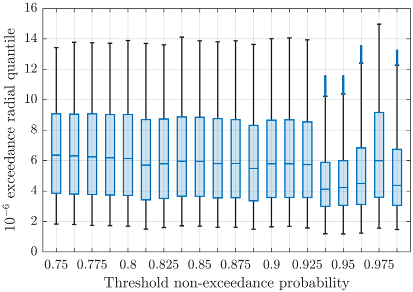
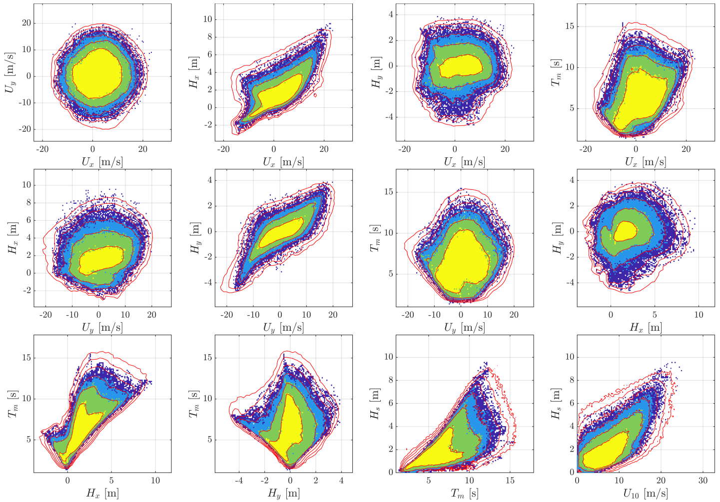


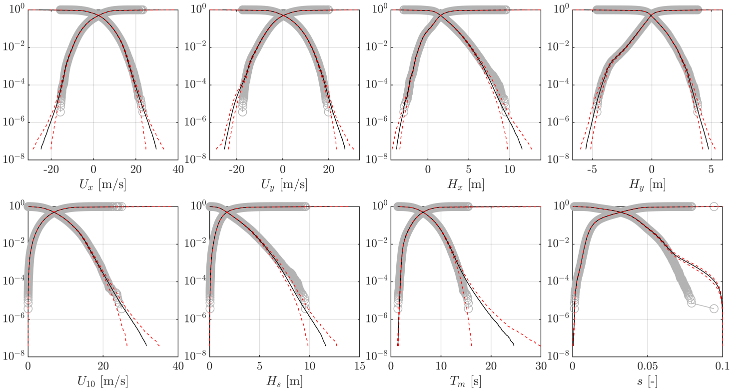
Figure: Box-plots of $\xi(\mathbf{w})$ estimates (left) and conditional radial quantile at exceedance level 10^{-6} (right).

Estimates



Contours produced from deepSPAR model using 3000 years of replicates.

Marginal diagnostics



Wind speed: $U_{10} = \sqrt{U_x^2 + U_y^2}$. Steepness: $s = 2\pi H_s/(gT_m^2)$.

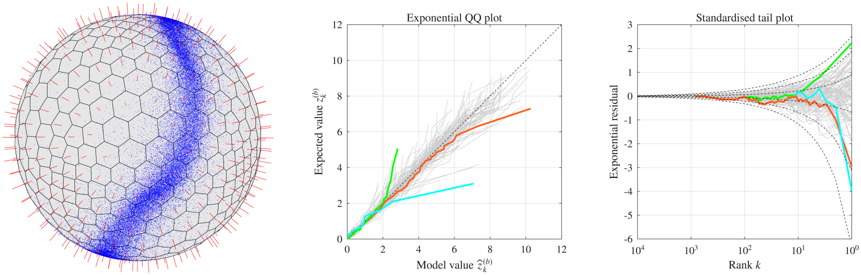
Conclusion

- Fast, flexible model for generating multivariate extremes in any direction with **minimal regularity assumptions**.
- **No marginal model** - just applied straight to observed data (subject to a choice of centre).
- Code available on GitHub:
  - callumbarltrop/DeepSPAR;
- New work looking at temporal changes in Alpine runfoff (Murphy-Barltrop et al., 2026).
- New diagnostic tools for extreme value regression models can help optimise architecture (Mackay et al., 2026).

Murphy-Barltrop, Richards, Poschlod, Sasse, Zscheischler (2026). Exploring climate change effects on concurrent floods and concurrent droughts via statistical deep learning. arXiv.

Mackay, Richards, Jonathan (2026). Diagnostic tools for extreme value regression models. arXiv.

Standardised Tail Plot

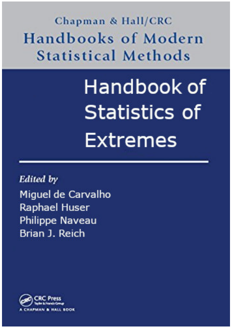


Left: Voronoi partition. Centre: Example simulations for 100 samples of random size N, where $\log_{10}(N) \sim \mathrm{U}(1, 4)$. Right: Same data transformed to standardised scale, together with quantiles of the asymptotic distribution at non-exceedance probabilities 0.001, 0.025, 0.25, 0.75, 0.975, and 0.999 (dashed lines).

Shameless advertisements

Gimeno-Sotelo, Richards, Hazra, Mhalla, and de Zea Bermudez. (2026).
A Review of Applications of Extreme Value Theory to Environmental Risk Assessment.
Environmental Modelling with Contemporary Statistics: Learning, Directionality, and Space-Time Dynamics, Chapman and Hall/CRC.

Shameless advertisements 2



Chapter 21. Richards and Huser (2026).
Extreme Quantile Regression with Deep Learning.
Code and short courses on GitHub - `https://github.com/Jbrich95/`

Shameless advertisements 3...

Home > Collection

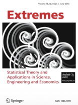
Bridging Heavy Tails and Artificial Intelligence

Participating journal: Extremes

Closed for submissions

The increasing frequency and severity of extreme events—such as catastrophic flooding, record-breaking temperatures, and unprecedented heatwaves—has highlighted the urgent need for innovative approaches in risk assessment and modeling. Modern advancements in data-collection techniques have provided increasingly large and complex datasets, which can only be processed used fast and scalable algorithms and computational software. This special issue aims to bridge the gap between Artificial Intelligence (AI) and Extreme Value Theory (EVT) to harness the strengths of both fields and address the growing challenges posed by these extreme events.

Participating journal



Journal

Extremes

Extremes is a dedicated platform for the publication of original research in statistical extreme value theory and its applications across various fields.

Publishing model	Hybrid
Journal Impact Factor	1.1 (2023)
Downloads	58k (2024)
Submission to first decision (median)	7 days

https://link.springer.com/collections/haghbdfdhb

References I

Mackay, E. and Jonathan, P. (2023). Modelling multivariate extremes through angular-radial decomposition of the density function. *arXiv preprint arXiv:2310.12711*.

Mackay, E., Murphy-Barltrop, C. J. R., Richards, J., and Jonathan, P. (2025). Deep Learning Joint Extremes of Metocean Variables Using the SPAR Model. *Journal of Offshore Mechanics and Arctic Engineering*, 148(021201).

Richards, J. and Huser, R. (2026). Extreme Quantile Regression with Deep Learning. In de Carvalho, M., Huser, R., Naveau, P., and Reich, B. J., editors, *Handbook on Statistics of Extremes*. Chapman & Hall/CRC, Boca Raton, FL.

Wessel, J. B., Murphy-Barltrop, C. J., and Simpson, E. S. (2025). Generative machine learning for multivariate angular simulation. *Extremes*, pages 1–49.

Fin.

SCAN ME

Scan for webpage.