Modelling the tail behaviour of precipitation aggregates using conditional spatial extremes

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Motivation

- River flooding caused by high intensity rainfall.
  Flooding $\rightarrow$ damage $\rightarrow$ cost
- Flooding not caused by rain at a single location, rather spatial aggregate over catchment area
- For spatial process $\{Y(s)\}$ observed at sampling locations (spatial sets) $s = (s_1, \ldots, s_d) \subset S$, interested in behaviour of aggregate $R_A = \frac{1}{|A|} \int_A Y(s)\mathrm{d}s$ for $A \subseteq S$ (or equivalent sum)
- Particularly interested in extremal behaviour (return levels) - largest events likely to be the most damaging
- Can just take sample aggregate R_A - use *GPD* modelling and extrapolate into tails

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- Wasteful - Orders of magnitude less data to work with
- Takes no information from the underlying marginal process or dependence structure $\rightarrow$ Could potentially lead to inconsistent inference i.e., no natural ordering of estimated return levels where physically appropriate
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- Model for marginal extremes and extremal dependence
- Focus on modelling extremes of process - largest values of underlying process produce largest values of aggregate
- Marginal model at each site -
  - *GPD* above threshold, empirical below
  - Spatial smoothing through GAMs [Youngman, 2019]
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Dependence model - general outline

General outline for spatial conditional extremes [Wadsworth and Tawn, 2019]

- Spatial process $\{X(s) : s \in S\}$ with standard exponential upper-tailed margins (standard Laplace)
- Condition on observing high value of process at conditioning site i.e., $X(s_0) > u$
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Dependence model

For locations $\{s_1, \ldots, s_d; d \in \mathbb{N}\}$, assume that there exists normalising functions $\{a_{s-s_0} : \mathbb{R} \to \mathbb{R}, s \in S\}$, with $a_0(x) = x$, and $\{b_{s-s_0} : \mathbb{R} \to (0, \infty), s \in S\}$, such that

$$\left(\left\{ \frac{X(s_i) - a_{s_i - s_0}\{X(s_0)\}}{b_{s_i - s_0}\{X(s_0)\}} \right\}_{i=1,\ldots,d}, X(s_0) - u \right) |X(s_0) > u$$

$$\xrightarrow{d} \left(\left\{ Z^0(s_i) \right\}_{i=1,\ldots,d}, E \right), \text{ as } u \to \infty,$$

- E as a standard exponential variable
- (Residual) process $\{Z^0(s) : s \in S\}$ independent of E, satisfies $Z^0(s_0) = 0$ almost surely

Modelling choices - Normalising functions

Assume limit holds for $X(s_0) > u$. We have

$$\{X(s)\} = a_{s-s_0}\{X(s_0)\} + b_{s-s_0}\{X(s_0)\}\{Z^0(s)\}.$$

Modelling choices for normalising functions:

- Let $a_{s-s_0}(x) = x\alpha(s-s_0)$ for $\alpha : \mathbb{R}_+ \to [0,1]$ and let $b_{s-s_0}(x) = x^{\beta(s-s_0)}$ for $\beta : \mathbb{R}_+ \to [0,1]$.
- As $a_0(x) = x$, require $\alpha(0) = 1$.
- To have $\{X(s)\}$ independent of $X(s_0)$ at large distances, need $\lim_{s-s_0\to\infty} \alpha(s-s_0) = \lim_{s-s_0\to\infty} \beta(s-s_0) = 0$.
- Control dependence strength - Extremal dependence between $X(s_i)$ and $X(s_0)$ decreases with $\alpha(s_i - s_0)$.

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Residual process - Dependence

Reminder that we have

$$\{X(s)\} = \alpha(s - s_0)X(s_0) + X(s_0)^{\beta(s-s_0)}\{Z^0(s)\}.$$

Modelling choice for dependence in $\{Z^0(s)\}$:

- Start with a standard GP $\{Z(s)\}$ with stationary correlation structure
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Modelling choice for margins of $\{Z^0(s)\}$:

- Set margins of $\{Z^0(s)\}$ to delta-Laplace, with density

$$f(z) = \frac{\delta}{2k\sigma\Gamma\left(\frac{1}{\delta}\right)} \exp\left\{-\left|\frac{z-\mu}{k\sigma}\right|^{\delta}\right\},$$

  with $\Gamma(\cdot)$ as the standard gamma function and $\mu \in \mathbb{R}, \sigma > 0, \delta > 0$ and $k^2 = \Gamma(1/\delta)\Gamma(3/\delta)$.
- More flexible than Laplace ($\delta = 1$) or Gaussian ($\delta = 2$) as allows for both.

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- Parametrise μ, σ and δ as functions of $s - s_0$.
- For $Z^0(s_0) = 0$ almost surely, require $\mu(0) = \sigma(0) = 0$.
- For standard Laplace margins at large enough distances - need $\mu(s - s_0) \to 0, \sigma(s - s_0) \to \sqrt{2}$ and $\delta(s - s_0) \to 1$ as $s - s_0 \to \infty$.

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Censoring

Biggest issue when modelling precipitation is the occurrence of dry periods (zeroes in the data) which make the lower marginal tails discrete.

- Cannot remove zeroes - Aggregates over larger spatial areas may contain zeroes
- Solution - Censored likelihood
  - Estimate $p(s) = \Pr\{\text{No rain as } s\}$ from data using a logistic GAM.
  - Set $c(s) = F_L^{-1}\{p(s)\}$ where $F_L(\cdot)$ is the standard Laplace CDF i.e. probability of no rain on standard Laplace scale
  - Use $c(s)$ as censoring threshold in likelihood. When simulating from dependence model, set any value below $c(s)$ to 0.

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Inference & Simulation

- Inference
  - Pairwise likelihood due to computational complexity - Calculate likelihood for single conditioning site
  - Want to simulate given extreme at any conditioning site - Assume stationary dependence - Take product over all conditioning sites
- Simulation - Full details omitted
  - Simulate $\{X(s)\}|\max_{s\in S} X(s) > u$ i.e. extreme at any conditioning site - Using importance sampling
  - Require aggregate of unconditional process - Use observations for $\{X(s)\}|\max_{s\in S} X(s) < u$
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- Hourly precipitation rate (mm/hour), Summer (JJA), 1980-2000
- From UKCP18 climate projections - values assigned to 934 spatial grid-boxes rather than point locations - Require mean for R_A, rather than integral
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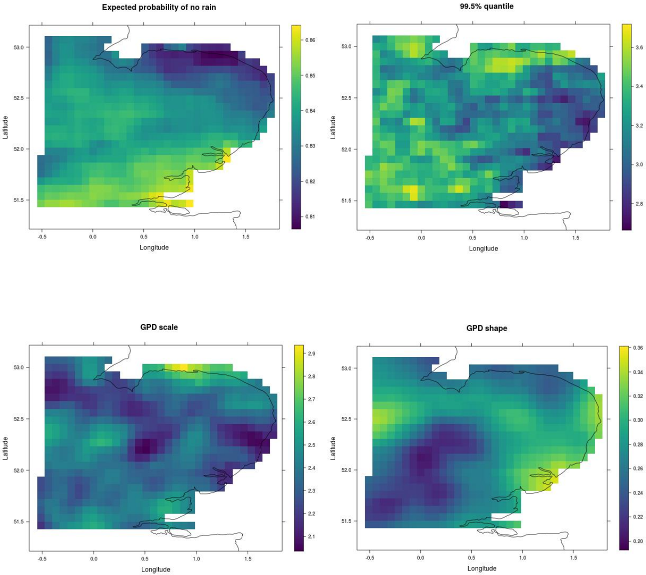
Marginal model results



Dependence Model - Parametric function choice

- To get an idea of the functional forms of the dependence parameters, we fit a simple dependence model
  - No dependence in residual process $\{Z^0(s)\}$
  - Individual parameter estimates, rather than fitted functions i.e. sequence of α_{s_i} for $i = 1, \ldots, d$, not $\alpha(s - s_0)$
  - Done for several conditioning sites spread out over domain
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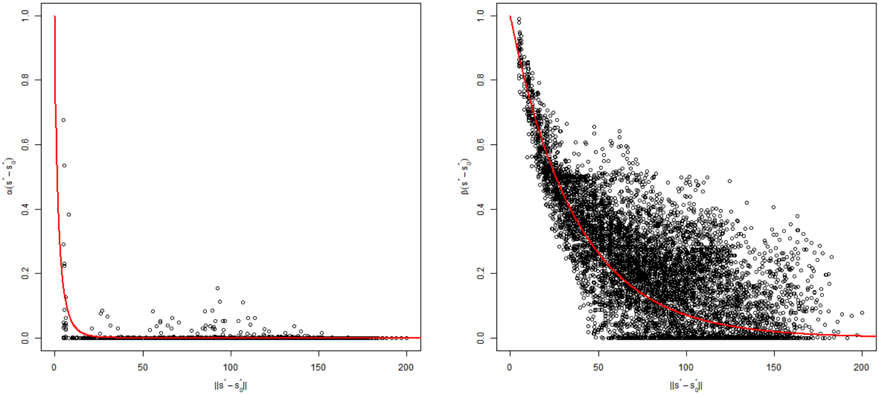
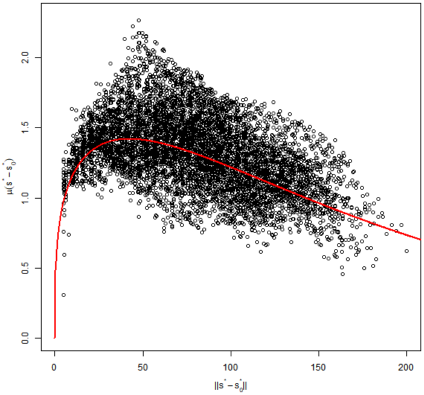
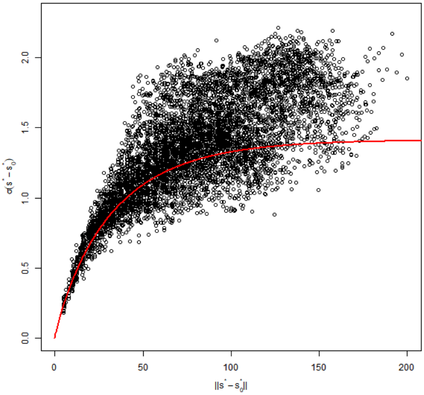
Alpha/Beta



- Powered exponential decay - $\exp(-((s - s_0)/\lambda)^{\kappa}), \quad \lambda > 0, \kappa > 0$

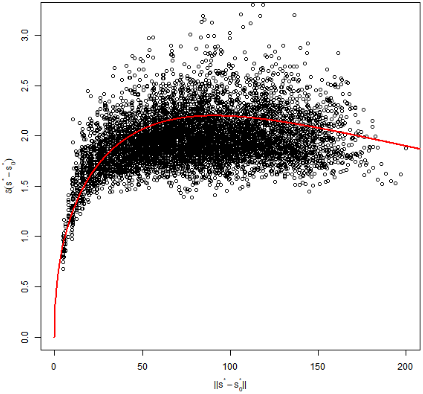
Mu/Sigma





- $\mu(s - s_0) = K(s - s_0)^{\kappa} \exp(-(s - s_0)/\lambda), \quad K > 0, \lambda > 0, \kappa > 0$ i.e. Gamma kernel
- $\sigma(s - s_0) = \sqrt{2}\{1 - \exp(-((s - s_0)/\lambda)^{\kappa})\}$ i.e. bounded powered exponential growth

Delta/Correlation

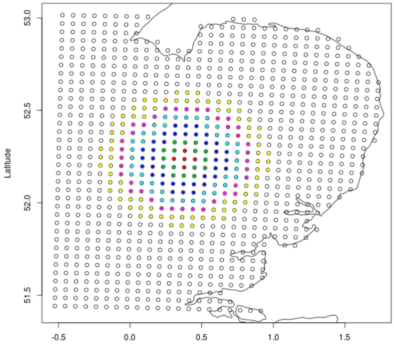


- $\delta(s - s_0) = 1 + (K_1(s - s_0)^{\kappa} - K_2)\exp(-(s - s_0)/\lambda), \quad K_1 > 0, K_2 < 1, \lambda > 0, \kappa > 0$ i.e. shifted-Gamma kernel
- Matérn correlation function (not pictured)

Aggregate diagnostics

Q-Q plots for high quantiles:

- How well does the model for $\{Y(s)\}$ replicate empirical R_A?
- Simulate over entire domain
- Aggregate over increasing larger regions (coloured points and interior) $(125, 525, 1425, 2425, 3350, 5425) - km^2$
- Compare quantiles against data



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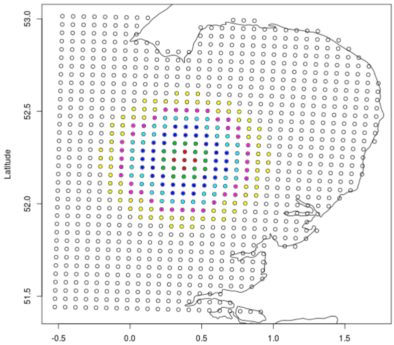

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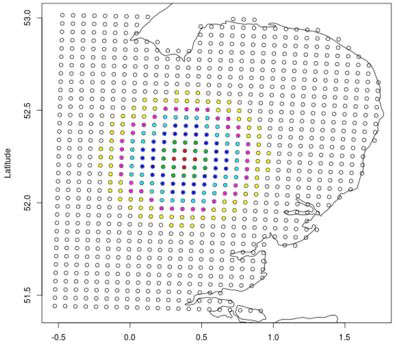

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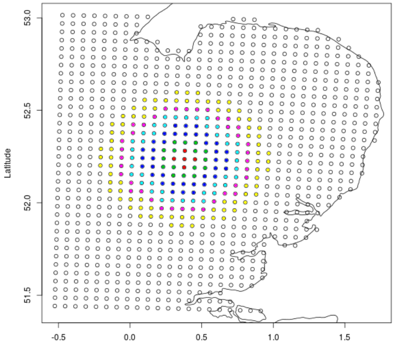
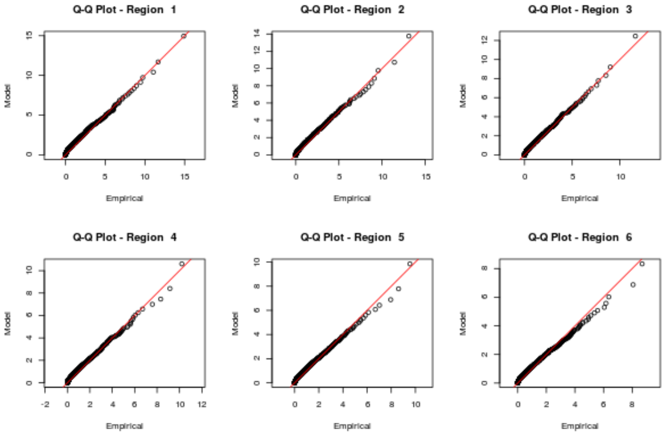


Model diagnostics

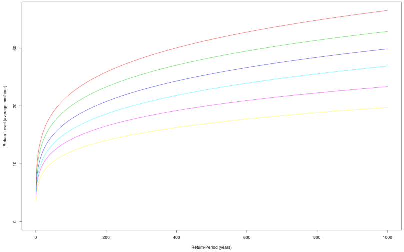
Q-Q plots for high quantiles:

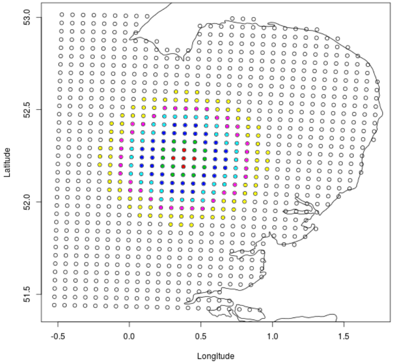
- Regions increase in size with label (i.e., 1 smallest - 6 biggest)
- Largest quantile corresponds to a 20 - year return level



Results

- *GPD* modelling for aggregate return levels - up to 1000 year return-level
- No bias-variance trade-off issues - can just simulate more realisations
- Self-consistent i.e. monotonically decreasing as (nested) region size increases





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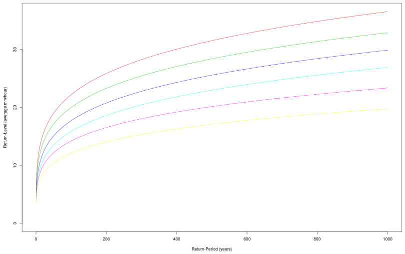

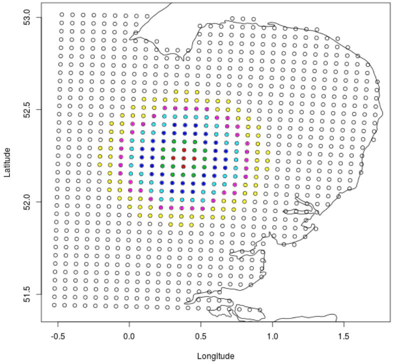

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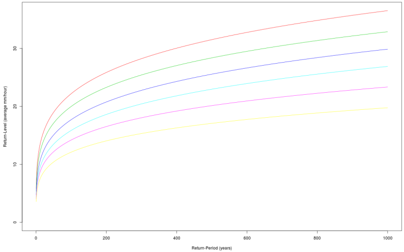

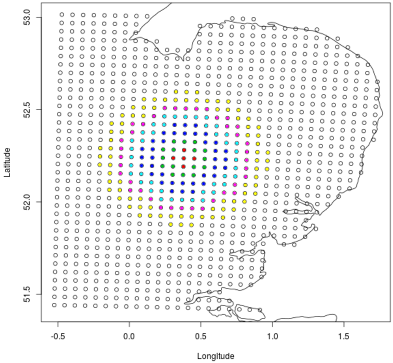

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Thanks for listening.

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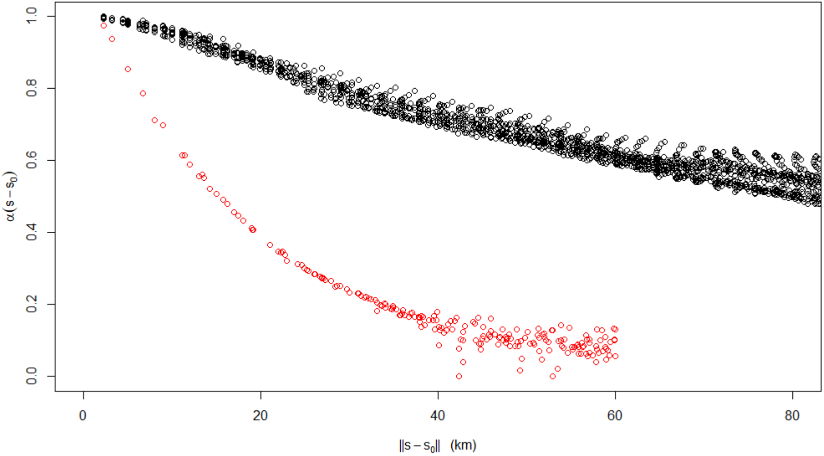
- $\alpha(\cdot)$ estimates for convective (red) and non-convective rain



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